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DETAILED ANALYSIS FOR PHOTOVOLTAIC POWERED WATER PUMPING SYSTEMS

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Abstract—In this work a model for each element as well as their interactions have been developed, [1–9]. The problem of obtaining the characteristics of the pump at different speeds beginning from the normally given nominal speed curve is explained. The power loss in the pump and motor are regarded in the model. The case of surface pumping from a river (constant head) has been considered too. The DC permanent magnet motor is selected due to its higher efficiency. A detailed example has been given to explain how to use the model. The DC permanent magnet motor centrifugal pump group is proved to be suitable for direct coupling with PV array in both cases of variable and fixed heads.

1. INTRODUCTION

Direct coupling of a DC motor to a photovoltaic (PV) array has been studied at different loads (namely ventilator and constant loads) by Appelbaum and co-workers [1–3]. In [2] the permanent magnet motor is considered while in [3] both series and shunt DC motors are regarded. The main conclusion of Appelbaum is that direct coupling of PV array to a DC motor induces better matching in the case of ventilator load type than that of constant load. The same result has been proven by Roger [4] theoretically. An analysis of photovoltaic powered water pumping system is given by Braunstein and Kornfeld [5] in which the case of DC series motor is considered. The relationship between input power and the head developed by the pump is derived in [5]. Leguerre and Lascand [6] have studied the daily behaviour of PV water pumping system composed of a centrifugal pump driven by a permanent magnet motor. Roger and co-workers [7] have compared the analytical results obtained by a simulation program to those obtained in the field. Although good agreement between the results of simulation program and experiments in [7] exist, the model used in that program has not been given. Franx [8] has discussed the pumping from deep wells, he proposed a system composed of an immersed motor pump group. Because of the brushes problem for DC motors, one has to use an AC motor. Consequently a DC/AC inverter is indispensible in such a system [8]. The adaptation of a pumping system using a volumetric pump has been discussed by the authors [9]. In such a case a DC/DC converter is necessary to improve the matching efficiency between the PV array and pumping system [9].

The objective of the model developed here is to obtain the daily, monthly and annual performances of the PV powered pumping system for given conditions.

(1) *Climate and site data*

(a) Monthly average daily solar energy received on horizontal surface H_B (kWh m^{-2} day^{-1}), (b) monthly average ambient temperature during day T_a, (c) monthly average wind speed W (m s^{-1}) and (d) latitude of the site (L).

(2) *PV array data*

(a) Open circuit voltage V_{oc}, (b) shift circuit current I_{sc}, (c) series and shunt resistances R_s and R_{sh} of the array, (d) thermal voltage of the array V_T and (d) reverse saturation current I_0.

(3) *Well data*

(a) Depth of the well H_0, (b) area of the well A_W and (c) permeability of the layer containing water K (m h^{-1}).

(4) *Motor data*

(a) Electromotive force constant K_e (V r.p.m.$^{-1}$), (b) torque constant K_t (N m A^{-1}), (c) the friction torque T_f (N m), (c) armature resistance R_a (ohms), (d) nominal voltage V_n and current I_n and (e) maximum allowed current.

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(5) *Pump data*

(a) Head discharge curve at nominal speed, (b) nominal head H_n (m), (c) nominal discharge Q_n, (d) nominal speed N_n and torque T_n(Nm), (e) the discharge–current relationship at nominal speed and (f) length L_c and diameter D_c of the column associated with the pump.

(6) *The monthly average load demand* Q_L (m^3 day^{-1})

This is an important parameter for determining the size of water storage reservoir. The climatological data should be properly manipulated to get the instantaneous global radiation G (kW m^{-2}) on the tilted array surface as shown below.

2. ESTIMATION OF SOLAR RADIATION ON TILTED SURFACE

Starting from the monthly average solar energy received on a horizontal surface H_B (kWh m^{-2} day^{-1}), Klein [10] has described a convenient method to obtain the monthly average values of solar radiation on a tilted surface H_T (kWh m^{-2} day^{-1}). The technique described by Klein [10] depends on determining both direct (H_D) and diffused (H_d) monthly average daily energies beginning from H_B. The clearness index K_T is defined by,

$$K_T = \frac{H_B}{H_{ext}} \tag{1}$$

where H_{ext} is the extra terrestial solar energy received on a horizontal surface at the same location. The relation between H_d and H_B is given by Page's formula [11],

$$\frac{H_d}{H_B} = 1 - 1{\cdot}13\, K_T. \tag{2}$$

Consequently the direct radiation H_D ($= H_B - H_d$) is simply

$$H_D = 1{\cdot}13\, K_T H_B. \tag{3}$$

If the PV array is directed towards the equator and tilted at an angle B then H_T is given by [10],

$$H_T = H_B[1{\cdot}13\, K_T R_b + 1/2(1+\cos B) \times (1 - 1{\cdot}13\, K_T) + 1/2\rho(1-\cos B)] \tag{4}$$

where R_b is the ratio between direct radiation on tilted and horizontal surfaces respectively and ρ is the reflectivity of the ground at the site where the PV array is installed. If there is snow $\rho = 0{\cdot}7$ otherwise $\rho = 0{\cdot}2$. The instantaneous global solar radiation on tilted surface G (kW m^{-2}) is given by Liu and Jordan's relationship [12],

$$G = \left(\frac{\pi}{24}\right) H_T \frac{\cos\omega - \cos\omega_s}{\sin\omega_s - \omega_s\cos\omega_s} \tag{5}$$

where ω and ω_s are the hour and sunset angles respectively [12].

3. OUTPUT OF PHOTOVOLTAIC ARRAY

Neglecting the shunt resistance of the PV array, the voltage–current relation of a PV array can be written as [2],

$$V_P = V_T \ln\left(1 + \frac{I_{sc} - I}{I_0}\right) - IR_s \tag{6}$$

where I is the load current. Both thermal voltage V_T and reverse saturation current I_0 depend on solar cell temperature T_c where for silicon [13],

$$(V_T)_{T=T_c} = (V_T)_{T=T_0}\left(\frac{T_c}{T_0}\right) \tag{7}$$

and

$$(I_0)_{T=T_c} = (I_0)_{T=T_0} \times 2^{(T_c - T_0)/10}. \tag{8}$$

The temperatures T_0 and T_c in equation (7) are in degrees Kelvin. The authors have studied the problem of estimating solar cell operating remperature [14] and it is shown that T_c can be given by [14],

$$T_c = \left[\frac{(U_L/F_p) - (m_0/F_p)(T_a + T_{c0})}{2(m_0/F_p)}\right] \times [(1+Y)^{1/2} - 1] \tag{9}$$

and

$$Y = \frac{4(m_0/F_p)\{(U_0/F_p)T_a + (\alpha - \eta'_R)G\}}{\{(U_0/F_p) - (m_0/F_p)(T_a + T_{c0})\}^2} \tag{10}$$

where U_0, m_0 and T_{c0} are constants for a given climate as explained in [14]. F_p is the module packing factor, α and η'_R are absorption coefficient and modified solar cell efficiency respectively [14]. U_L is the heat transfer coefficient (W m^{-2} °C^{-1}) which depends on wind speed and sky temperature. Simpler expressions to determine T_c given by Seigel [15] and by Evans [16] lead to overestimated values for T_c as explained by the authors [14]. The short circuit current I_{sc} in equation (6) is directly proportional to global instantaneous solar radiation G (kW m^{-2}),

$$I_{sc} = I_{sc0} \cdot G \tag{11}$$

where I_{sc0} is the short circuit current at AM1 conditions.

4. THE WELL CHARACTERISTICS

At steady state conditions, the water depth of a given geological composition is constant at H_0 as shown in Fig. 2. When a well with radius r_0 is dug through the layer to pump water at a rate Q (m^3 h^{-1}) up to the earth's surface, then the water surface will decline from its equilibrium state (at H_0 level) to reach its minimum level at the edges of the well as shown in Fig. 1. If K is the permeability of the layer containing water (m h^{-1}) then the discharge Q can be written using Darcy's equation,

$$Q = K\cdot(2\pi rh)\frac{dh}{dr} \tag{12}$$

where $2\pi rh$ is the bilateral area of the water in the well if its radius is r. From Darcy's expression one can write

$$Q\int_{r0}^{R}\frac{dr}{r} = 2\int_{h}^{Z} K\pi h\, dh$$

where Z is thickness of the water layer. Performing integration one gets,

$$Q = \pi K\frac{Z^2-h^2}{\ln(R/r_0)}. \tag{13}$$

The depth of the well h_w can be seen from Fig. 1 to be,

$$h_w = H_0+(Z-h).$$

Using equation (13) to eliminate h one can write,

$$h_w = H_0+Z[1-\sqrt{1-C_WQ}] \tag{14}$$

where

$$C_W = \frac{\ln(R/r_0)}{\pi KZ^2}. \tag{15}$$

C_W is a well parameter which is only affected by the well radius r_0. Figure 2 shows a schematic plot for equation (14). Since the pumping takes place only when the solar radiation is available, then it is expected that the well will recuperate during the evening and night times and consequently $h_W = H_0$ at early morning every day. The

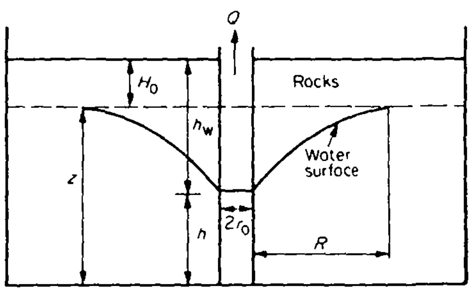

Fig. 1. Schematic diagram of a typical well.

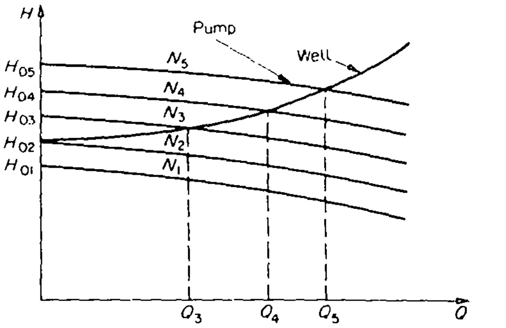

Fig. Well depth vs discharge, and H–Q characteristics of both well and centrifugal pump.

value of H_0 is a function of the season. In winter time, H_0 decreases due to the rain falling, while during the summer dry season H_0 is expected to increase.

5. HEAD LOSS IN THE PUMP COLUMN

The head loss H in the pump column assuming turbulent flow and rough pipe—which are the common practical situation—is given by [17],

$$\delta H = 0{\cdot}3164\frac{(R_e)^{-0{\cdot}25}}{\rho_f g}\left(\frac{L_c}{D_c}\right)Q^2 \tag{16}$$

where L_c and D_c are the column length and diameter respectively. R_e is Reynold's number for the flow (generally exceeds 10,000 for turbulent flow) and ρ_f is the density of the fluid. The head to be developed by the pump should be equal to the sum of the well depth h_w and the head loss in the column and the head of the reservoir where the water is stored (if any).

6. DC PERMANENT MAGNET MOTOR

The centrifugal pump is generally coupled to a permanent magnet DC motor which is fed by a PV array. Contrary to the conventional power supplies, the solar energy and consequently the output power of the PV array vary continuously. Therefore both current and voltage of the DC motor change from one instant to the other. The torque of a DC permanent motor T is given by,

$$T = K_t I - T_f \tag{17}$$

where I is the motor current, T_f is the friction torque and K_t is a constant. This is a straight line equation. The current I_1 is the minimum current required to be supplied to the motor to start revolving (i.e. the torque T_f). The voltage across motor terminals is equal to that

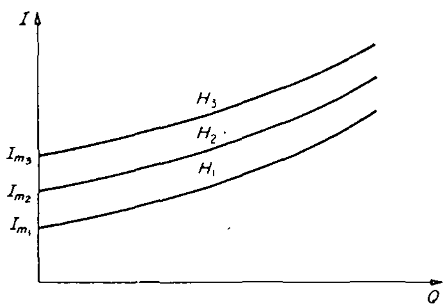

Fig. 3. I–Q relation for different heads.

across PV array and is given by,

$$V_M = IR_a + K_e N \tag{18}$$

where N is the speed, R_a is the armature resistance and K_e is the electromotive force constant. For a constant voltage, a linear relation between I and N is obtained. From such linear relation one can determine both R_a and K_e. If I_n and N_n are the nominal current and speed respectively and N_0 is the maximum speed of the motor, then one can write,

$$K_e = \frac{V_M}{N_0} \tag{19}$$

and

$$R_a = \left[1 - \left(\frac{N_n}{N_0}\right)\right]\left(\frac{V_M}{I_n}\right). \tag{20}$$

7. CENTRIFUGAL PUMP

Figure 2 indicates the H–Q relationship of a typical centrifugal pump at different speeds ($N_5 > N_4 > N_3 > N_2 > N_1$). The H–Q curve of the well is imposed on that of the pump in Fig. 2. The H–Q relation of a centrifugal pump can be approximated by [5],

$$H = (H_0 - K_1 Q)(1 - K_2 Q^2) \tag{21}$$

where K_1 and K_2 are constants for a given pump and they can be deduced from the given H–Q curve of the pump. H_0 is the maximum head that can be developed at a given speed. As speed N increases the head increases too according to [18],

$$\frac{H}{H_n} = \left(\frac{N}{N_n}\right)^2 \tag{22}$$

where H_n and N_n are the nominal head and speed respectively. If the pump speed in Fig. 2 is N_1 then for the shown well characteristics, no discharge may take place because the head developed by the pump is less than the well head. In such a case the motor runs but without discharge obtained from the pump. The mechanical energy of the motor at a speed N_1 will be totally converted into heat which induces the water temperature increase. The minimum speed required to start discharging is N_2 at which H_{02} is just equal to h_w. For higher speeds such as N_3, N_4 and N_5 the discharge is determined by the intersection of pump and well characteristics as shown in Fig. 2.

The minimum current I_m required to start discharge depends on the head required as shown in Fig. 3. Since the torque of the centrifugal pump (which equals to the motor torque) is proportional to the square of the speed [5] then,

$$\frac{T}{T_n} = \left(\frac{N}{N_n}\right)^2. \tag{23}$$

Using equations (22) and (23) one finds,

$$\frac{T}{T_N} = \frac{H}{H_n} \tag{24}$$

where T_n is the nominal torque. If the minimum current required to have discharge at nominal head is I_{mn}, then from equation (17) the minimum torque needed to obtain discharge at the nominal head T_{mn} is,

$$T_{mn} = K_t I_{mn} - T_f. \tag{25}$$

From equations (24) and (25) the minimum torque T_m required at head H can be written as,

$$T_m = T_{mn}\frac{H}{H_n} = (K_t I_{mn} - T_f)\cdot\frac{H}{H_n}. \tag{26}$$

Since the minimum current I_m at head H can be written in terms of T_m using equation (17) then

$$T_m = K_t I_m - T_f. \tag{27}$$

Eliminating T_m between equations (26) and (27) one

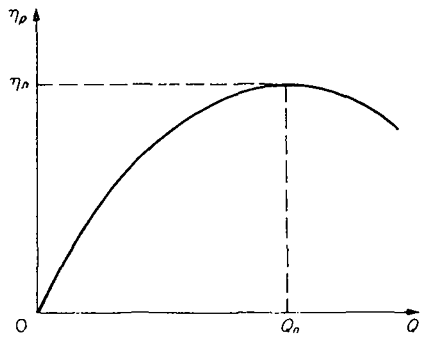

Fig. 4. Pump efficiency versus discharge.

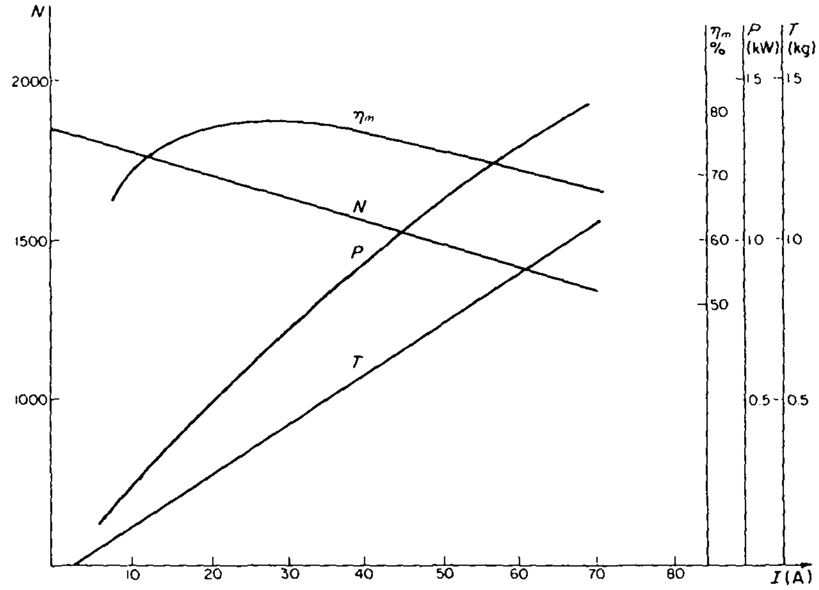

Fig. 5. Characteristics of DC permanent motor.

finds,

$$I_m = I_{mn}\left(\frac{H}{H_n}\right) - \left(\frac{T_f}{K_t}\right)\left(\frac{H}{H_n} - 1\right). \tag{28}$$

If the motor current I is less than I_m at a given head H then there will be no discharge ($Q = 0$) and the overall system efficiency would be zero in spite of the rotation of the motor. The current I_m corresponds to the speed N_2 in Fig. 3 at which the system starts discharging. The efficiency of the pump η_p is given by,

$$\eta_p = \frac{vQH}{2\pi TN} \tag{29}$$

where v is the specific weight of the liquid to be pumped from the well.

From equations (21) and (29) one can write,

$$\eta_p = \frac{v}{2\pi TN} Q(H_0 - K_1 Q)(1 - K_2 Q^2). \tag{30}$$

For a given input mechanical power, equation (30) indicates that Q has a specified value at which η_p is maximum as shown in Fig. 4. Normally the nominal discharge Q_n is that which induces maximum pump efficiency η_n as shown in Fig. 4. At other operating

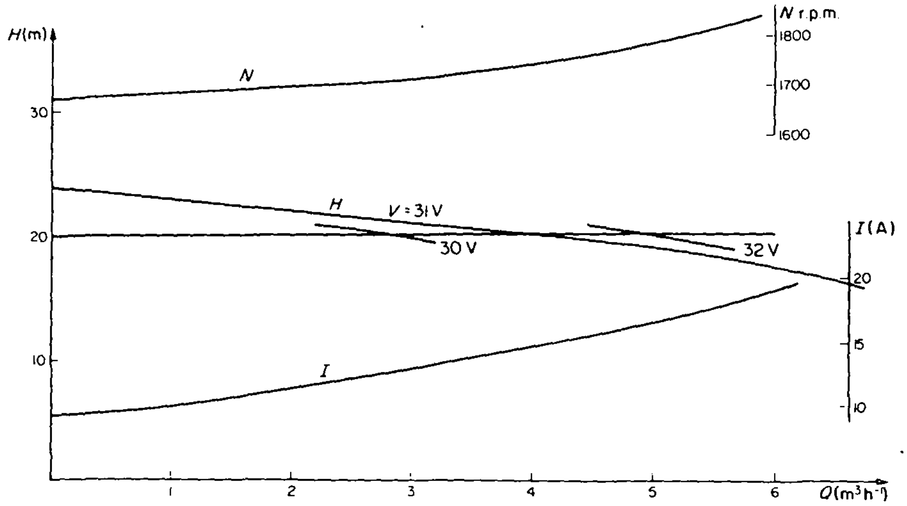

Fig. 6. Centrifugal pump characteristics.

points the pump efficiency can be related to η_n through,

$$\eta_p = \eta_n \frac{QH}{Q_n H_n}. \quad (31)$$

Combining equations (30) and (31) one can eliminate η_p,

$$(H_0 - K_1 Q)(1 - K_2 Q^2) = \left(\frac{2\pi\eta_n}{vQ_n H_n}\right) TNH. \quad (32)$$

For a given head H, the discharge Q can be obtained by iterative method from equation (32) if both torque T and speed N are known. At a given solar radiation G (kW m^{-2}) the torque and speed can be known if the head is known as it is explained below. Note that H_0 in equation (32) is a function of speed N according to equation (23) thus,

$$\frac{H_0}{H_{0n}} = \left(\frac{N}{N_n}\right)^2 \quad (33)$$

where H_{0n} is the maximum head developed at the nominal speed N_n. Generally one has to be careful to operate as closely as possible to the nominal operating point specified by manufacturer to get maximum efficiency. If the head is changing—as usual—then the nominal head H_n should be situated at about the average value of the head values as will be seen in the example given in later section.

8. COUPLING OF PUMP MOTOR GROUP TO PV ARRAY

In the case of direct coupling the array voltage V_p given by equation (6) is equal to the motor voltage V_M given by equation (18), thus one can write,

$$V_T \ln\left(1 + \frac{I_{sc} - I}{I_0}\right) - IR_s = IR_a + K_e N. \quad (34)$$

Combining equations (11), (22) and (34) one gets,

$$V_T \ln\left[1 + \left(\frac{I_{sc0}G - I}{I_0}\right)\right] = I(R_a + R_s) + K_e N_n \sqrt{\frac{H}{H_n}}. \quad (35)$$

For a given head H and insolation G, equation (35) can be solved numerically to get the value of the current I. Once the current I is known, the voltage V can be obtained from equation (6) then the speed can be obtained from equation (18). The torque T can be found from equation (17) since I is known from equation (35). Since the speed is known then H_0 can be obtained from equation (33) and afterwards the discharge Q may be obtained by the iteration method from equation (21).

The only condition to be verified before going through the described algorithm for obtaining the parameters of the system is that the current obtained by solving equation (35) is higher than I_m given by equation (28) for the same head H. The matching efficiency η_{mt} between PV array and pumping system is,

$$\eta_{mt} = \frac{IV}{P_{max}(G)} \quad (36)$$

where $P_{max}(G)$ is the array maximum power at insolation G. The mechanical efficiency η_M of the motor pump group is,

$$\eta_M = \frac{2\pi TN}{IV}. \quad (37)$$

The pump efficiency is given by equation (29). The global efficiency η_G is simply given by,

$$\eta_G = \frac{vQH}{P_{max}(G)} = \eta_{mt}\eta_M\eta_p. \quad (38)$$

9. EXAMPLE

Figures 5 and 6 show the characteristics of DC permanent magnet motor and a typical three stage centrifugal pump respectively. It should be recognized that for all curves of Fig. 5 the voltage is constant and equal to the nominal value which is 31 V in this example. In Fig. 6 for each curve all other parameters are constants and equal to their nominal values, for instance I–Q curve is drawn at the nominal head and nominal voltage. From Fig. 5 one can deduce the following constants for the DC motor; $K_e = 0.0167$ V r.p.m.$^{-1}$, $K_t = 0.0153$ N m A^{-1}, $T_f = 0.043$ N m, $R_a = 0.12$ ohm, $V_n = 31$ V, $I_n = 21$ A and $T_n = 0.28$ N m (corresponding to $I = I_n = 21$ A).

From Fig. 6 the following pump constants can be found; $Q_n = 6.4$ m^3 h^{-1}, $H_n = 17$ m, $H_{0n} = 24$ m, N_n

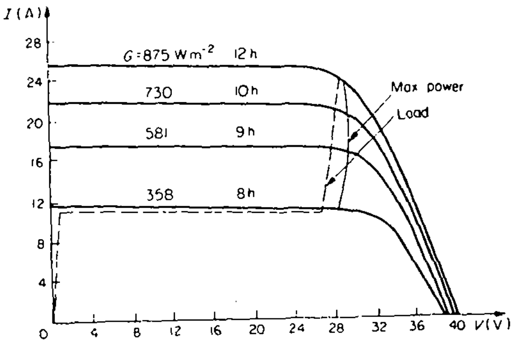

Fig. 7. Maximum power line and load line for variable head system.

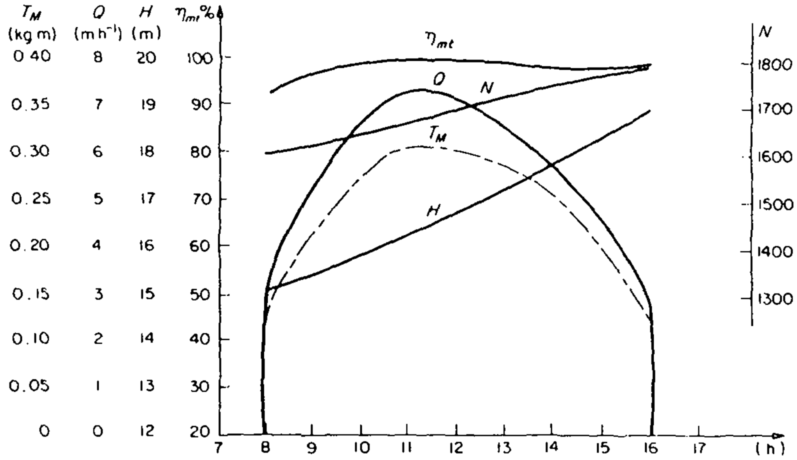

Fig. 8. Hourly response of a system with variable head.

= 1700 r.p.m., $K_1 = 0{\cdot}79$, $K_2 = 0{\cdot}003$ and $I_{mn} = 9$ A. Pump column has $L_c = 20$ m and $D_c = 0.15$ m.

The nominal discharge Q_n is obtained from I–Q curve at $I = I_n = 21$ A and then the nominal head H_n is obtained from the H–Q curve at $Q = Q_n$. Nominal speed corresponds to $I = I_n = 21$ A for N–I straight line indicated in Fig. 5. The minimum current I_{mn} is obtained from I–Q curve in Fig. 6 at $Q = 0$.

The PV array is composed of two modules in series and 12 strings in parallel (i.e. total number of modules is 24). Each module has the following characteristics at 25°C: $V_{0c} = 20$ V, $I_{sc0} = 2{\cdot}5$ A, $I_0 = 1{\cdot}88 \times 10^{-10}$ A, $V_T = 0{\cdot}858$ V and $R_s = 1{\cdot}5$ ohms.

The well characteristics are: $R = 12{\cdot}57$ m, $K = 1$ m h^{-1} and $H_{0w} = 15$ m.

The climate of Cairo city, Egypt has been considered as an example of tropical climate regions. The latitude of Cairo is 30°N and the annual average daily energy received on a horizontal surface is about 5·6 kWh m^{-2} day^{-1}. The climatological data of Cairo are obtained from [19].

A simulation program has been developed to study the behaviour of the pumping system. Two cases have been considered, (1) variable head as the typical characteristics of a well, (2) constant head where the water level is independent of the pumped water (case of river or large canal).

Figures 7–9 indicate the response of a system with varying head. The nominal head of the pump considered is 17 m, so the head variations should be centered around the nominal value. In the considered case H changes from 15 to 19 m as shown in Fig. 8. The matching efficiency between the pumping load and PV array is quite good as can be seen by Fig. 7. The system

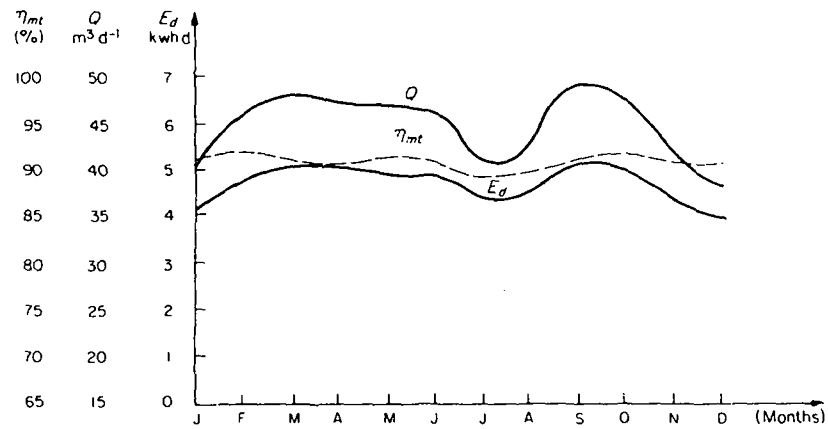

Fig. 9. Monthly average daily values of produced energy by PV array (E_d), discharge (Q) and matching efficiency for variable head system.

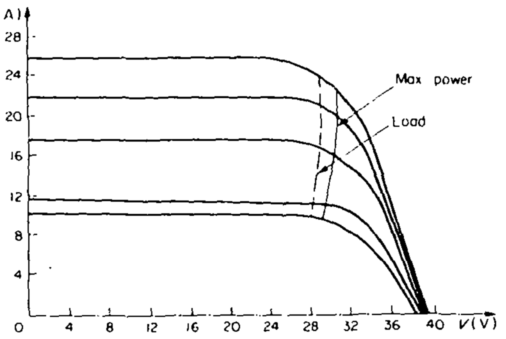

Fig. 10. Maximum power and load line for constant head system.

starts discharge at about 8 h with a minimum value of η_{mt} and it increases to about 100% at noon time as shown in Fig. 8. But it is seen from Fig. 8 that the matching efficiency always exceeds 90% during the operating period of the system. It can be seen from Figs. 8 and 9 that the motor speed is proportional to the head. The discharge varies nearly as the solar radiation as seen in Fig. 8. Both motor torque and current vary like the solar radiation as indicated in Fig. 8. The mechanical efficiency of the motor η_M is almost constant, but it only drops at low solar radiation because the friction losses represent a relatively large proportion of the input power.

The monthly average values of daily discharge, matching efficiency and energy produced by PV array E_d (kWh day^{-1}) are given in Fig. 9. It is seen that the matching efficiency varies between 89 and 92%. Thus the annual matching efficiency exceeds 90%. The high matching efficiency proves that the direct coupling between pumping system and PV array is favourable due to its simplicity, low cost, high efficiency and reliability.

Figures 10–12 show the performance of the pumping system when the head is constant and equal to the nominal head. Contrary to the case of variable head, the speed is constant because of fixed head. The motor torque, current and discharge are charging according to the solar radiation. It can be seen that the direct coupling of pumping system with PV array is acceptable in both cases of variable and fixed head, only in the case of variable head the mean value of the head should be about the nominal head.

It should be noted that the centrifugal pump when running at a constant speed does not deliver liquid at a fixed unvarying rate irrespective of the head. On the contrary the rate of flow is almost wholly dependent on the pressure or resistance in the rest of the system. If the resistance is considerable, the discharge will be relatively small; as the resistance diminishes, the discharge increases [20]. This holds true as long as the input power is kept constant. For PV powered pumping system the input power changes instantaneously and so for a fixed resistance (fixed head), the discharge changes, likewise the solar radiation as seen in Fig. 11. It can be seen from Figs. 11 and 12 that the speed is totally controlled by the head; for constant head the speed is constant too, while the speed increases with the head.

10. CONCLUSIONS

A computer model for the detailed interactions of a PV powered pumping system has been presented. A

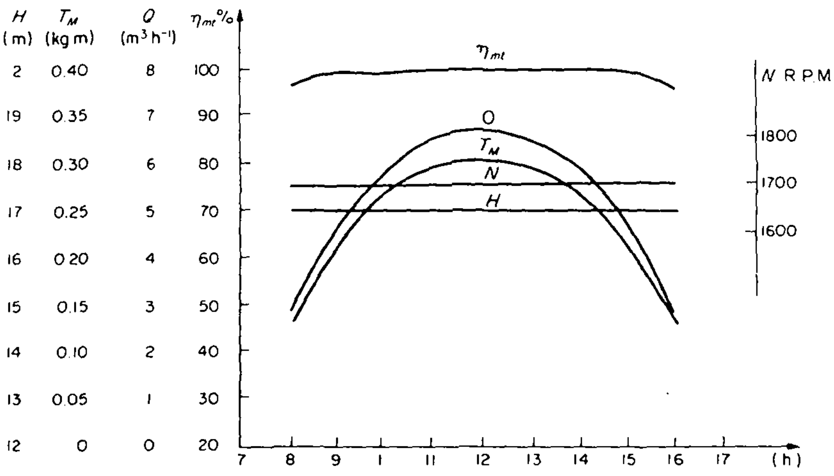

Fig. 11. Hourly response of constant head system.

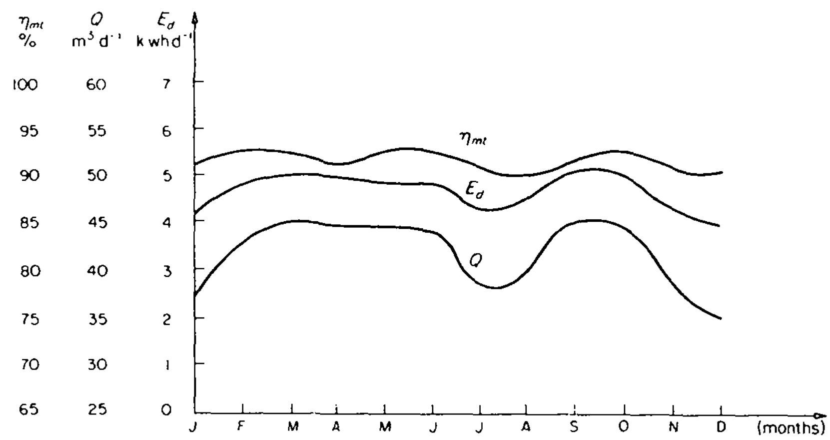

Fig. 12. Monthly average values of daily produced energy by PV array discharge and matching efficiency.

model for each element of the system (PV array, DC permanent magnet motor, centrifugal pump and the well) is given. The algorithm used to obtain the instantaneous, daily, monthly and annual performances has been explained. An example to clarify the use of the model is given. The performances of both constant and variable head systems have been considered and compared. DC permanent magnet motor-centrifugal pump group has a very good matching characteristics with PV array in case of direct coupling for both constant and variable head systems.

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